MATHEMATICS EIPIDEMIOLOGYON THE DYNAMICS OF COVID-19

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Abstract
In this paper, we aim to estimate and predict the situation of the new coronavirus pandemic (COVID-19) in countries under quarantine measures. First, we present a new discrete-time mathematical model describing the evolution of the COVID-19 in a population under quarantine. We are motivated by the growing numbers of infections and deaths in countries under quarantine to investigate potential causes. We consider two new classes of people, those who respect the quarantine and stay at home, and those who do not respect the quarantine and leave their homes for one or another reason. Second, we use real published data to estimate the parameters of the model, and then, we estimate these populations in India. We investigate the impact of people who underestimate the quarantine by considering an optimal control strategy to reduce this category and then reducing the number of the population at risk in India. We provide several simulations to support our findings.

Key Words: Covid, estimate, evolution, findings, number, potential, symptoms,

INTRODUCTION

Coronaviruses (CoV) are a large family of zoonotic viruses, that is, they are transmitted from animals to humans, and that cause symptoms ranging from the common cold to more serious illnesses such as Middle East Respiratory Syndrome (MERS) which is transmitted from dromedary to humans and severe acute respiratory syndrome (SARS) which is transmitted from civet to humans. Several known corona- viruses that have not yet infected humans are circulating in some animals [1].

The new coronavirus SARS-CoV-2 is a new strain of coronaviruses that have not yet been identified in humans, where respiratory symptoms, fever, cough, shortness of breath, and dyspnea, are common signs of the infection with the new coronavirus pandemic, COVID-19. In more severe cases, this disease can cause pneumonia, severe acute respiratory syndrome, kidney failure, and even death [1].

The WHO standard recommendations to the general public for reducing exposure to, and transmission of, a range of diseases are the following and include hand hygiene, respiratory hygiene, and good food safety practices [1]:

(i) Wash your hands frequently with a hydroalcoholic solution or with soap and water
(ii) Cover your mouth and nose with the crease of your elbow or with a handkerchief if you cough or sneeze; throw the handkerchief immediately after and wash your hands
(iii) Avoid close contact with people who have a fever and cough

(iv) In case of fever, cough, and difficulty in breathing, consult a doctor without delay and tell him the trips made.
(v) In markets located in areas where there are currently cases of the new coronavirus, avoid unprotected direct contact with live animals and with surfaces in contact with animals
(vi) Consumption of raw or undercooked animal products should be avoided. In accordance with good food safety practice, raw meat, milk, or organ meats should be handled with care to avoid cross-contamination with raw food.

At the moment, there is no vaccine for the new coronavirus SARS-CoV-2, because in general, when a new disease appears, a vaccine only becomes available after a vaccine development process, which can take several years. Further-more, there is no specific treatment for the disease resulting from the new coronavirus. However, many symptoms can be treated, and treatment therefore depends on the clinical condition of the patients. In addition, providing supportive care to infected people can be very effective [2].

Countries all over the world are intensifying their effortsto combat the COVID-19, which first appeared in the Chinese city of Wuhan, and that has killed at least 100,000 people. More than 1.5 million cases have been reported worldwide, most of them in the United States, Italy, Spain, and China. A man from Wuhan died in the Philippines on February 2, and Hong Kong reported the first death on February 4. The infection is now more widespread than the 2002-2003 severe acute respiratory syndrome (SARS) epidemic, also of Chinese origin, in terms of people affected and deaths.

Outbreaks of novel virus infections among people are always of public health concern, especially when there is little knowledge about the characteristics of the virus, how it spreads between people, how severe are the resulting infections, and how to treat them.

The mathematical modeling is one of the most solutions that can respond to all these questions. In the absence of vacancines or antivirals for the COVID-19, the effective implementation of nondrug interventions, such as personal protection and social exclusion, will be crucial in controlling the epidemic [3–5].

From the point of view of additional resources to fight the epidemic, mathematics and statistical modeling can be useful in creating forecasts for the cases announced. These expectations may include estimates of the burden of illness which can help guide public health officials in preparing for medical care and other resources needed to combat the epidemic. Predictions can also guide the severity and the type of interventions needed to mitigate the epidemic.

In this contribution, we develop a mathematical model that describes the evolution of the COVID-19 in countries under quarantine. Here, we use actual data from March 19 to April 05, 2020, of daily updates laboratory-confirmed cases, recovered, and deaths in India from the Johns Hopkins University Center for Systems Science and Engineering (JHU CCSE) [6], in order to estimate the parameters of the model and then predict the severity of the possible infection in the coming months. Using this method, we can estimate the total population at risk in India and justify the growing number of new confirmed cases despite the nationwide quarantine. Finally, we investigate an optimal control strategy that aims to reduce the population at risk in India, whereas the control strategy we are proposing here adopts nondrug intervention which makes it very optimal and more practical to use in developing countries.

The paper is organized as follows: Section 2 introduces our new model, giving some details about interactions between the different compartments and parameters of the model. In Section 3, we provide the results of the parameters'estimation validating our model and predicting the total population at risk in India. In Section 4, we present the optimal control problem to justify the existence and the characterization of the optimal control. Numerical simulations are carried out in Section 5. In Section 6, we provide a global sensitivity analysis, and Section 7 concludes.

2. Methods

2.1. Background. In late December 2019, several local health facilities reported groups of patients with pneumonia for unknown reasons linked to the seafood and wholesale wet animal market in Wuhan, Hubei Province, China. On December 31, 2019, the Chinese Center for Disease Control and Prevention (CDC) dispatched a rapid response team to support health authorities in Hubei Province and Wuhan City and conduct an epidemiological investigation to locate the source of pneumonia groups. This led to a description of a new coronavirus found in samples of pneumonia patients at the beginning of the epidemic [7]. Less than a month after identifying patients, on January 1, 2020, the government closed this seafood market [8]. A consortium led by a Chinese scientist and international researchers quickly gathered to intensify national and international efforts and coordination. On January 10, 2020, it released a partialsequence obtained from a patient from Wuhan who presented at least 70% similarity in genetic material to severe acute respiratory syndrome (SARS) [8].

Based on previous experiences with respiratory infections, the Chinese hospitals recognized the utmost importance of infection control preparedness in their healthcare system. Standby level includes alert, dangerous level 1, dangerous level 2, and emergencies; the activation level is deter- mined according to the risk assessment. Infection control measures and administrative support are strengthened with an indication of different levels of preparedness [9]. Through this infrastructure, China has overcome the challenge of influenza A pandemic in 2009 [10, 11] and the emergence of bird flu A H7N9 in 2013 [12].

To prepare for this emerging infectious disease, feverscreening was done at the airports and at the high-speed train stations, with special emphasis on flights and trains from Wuhan, while passengers with a fever above 38°C are referred to public hospitals for assessment. In addition, several control measures are implemented in the public hospital system [9].

Many countries have taken advantage of the experience of China in fighting this epidemic. This saved them time to try to contain the spread of the COVID-19 infection and learn them that this spare time must be used efficiently and effectively. India is one of these countries, whereas India was proactive in declaring a health emergency on March 20, at 6:00 pm until further notice, in response to the growing threat of a new coronary virus (COVID-19), after the

Ministry of Health confirmed case 63 of COVID-19, two patients recovered and two died. The state of emergency means that citizens cannot go out to public places without special permission from the local authorities. The state of emergency allows the government to take tougher measures to respond to the epidemic and to release budgets to improvemedical care to impose closures on whole cities. When all international flights to and from Moroccan territory are suspended until further notice, all public gatherings of more than 50 people are prohibited, including economic and political meetings, cultural and religious ceremonies, sporting events, and other gathering places closed. Courses have beenevents from time step i to time step i +1 as events at the same time. In our model, we choose the unit of time as a day, since COVID-19 data is collected daily in almost all countries of the world.

2.2. The Model. Despite all of the efforts made by the authorities, the number of confirmed COVID-19 cases is still rising, which means that something is wrong. One possible reason isthe lack of respect for the national quarantine and the under-estimation of the contagiousness of the SARS-CoV-2.

In the literature, over the past three months, several works of mathematical modeling have been published taking into account the exposed and asymptomatic classes [15–19]. This motivated us to consider more general classes in order to study the efficiency of the national closure and to evaluate our strategy of control that will be applied to all people regardless if they are exposed or not.

The first class is the aware individuals who know about

(i) S: the number of susceptible people to infection or who are not yet infected, and people that did not benefit from the awareness program

(ii) P: the number of susceptible people who are partially controlled. People who create arguments to leave their homes during the quarantine and can, there-size of the population at the instant i. Without loss of generality, we neglect the recruitment rate and the natural death rate due to the restricted time window of the study. $\theta = \theta 1 + \theta 2$ defines the awareness control parameter, where $\theta 1$ isthe recruitment rate of susceptibles to the partially controlledclass and $\theta 2$ is the recruitment rate of susceptibles to the totally controlled class. a is the inhibition effect due to resource limitation to awareness controls, β is the infectiontransmission rate, with "c" modeling the reduced chances of a partially controlled individual to be infected where $0 \leq c \leq 1$, T is the treatment rate, α is the infection death rate, $\gamma$is the recovery rate, and ρ is the losing removal individuals' immunity rate.

We note that the population size Ni is not constant in time i, in fact

$Ni+1 = Si+1 + Pi+1 + Ci+1 + Ii+1 + Ri+1$,fore, be considered as people not respecting the quarantine

(iii) C: the number of susceptible people who are totally

(iv) I: the number of infected people who are capable of spreading the epidemic to those in the susceptible and partially controlled categories

(v) R: the number of recovered people from the epidemic, but can return to the susceptible class because of the short-term removal of individuals' immunity
(vi) D: the death toll from COVID-19
In discrete-time models, we can choose the unit of time we need, a day, a week, or a month, and we collect all the data.

Estimation of the Parameters

Johns Hopkins University Center for Systems Science and Engineering (JHU CCSE) provides available data for academic research purposes from multiple sources [6]. Data information includes cumulative confirmed cases, the cumulative number of cases recovered, and the cumulative number of deaths. To get an insight into the evolution of the COVID-19 in India, we have acquired updated data on the cumulative number of laboratory-confirmed cases for COVID-19 from the CSSE data of [6] to estimate the parameters of the model (1), (2), (3), (4), (5) and (6). For more details about the available data, see [6].

The following process has been followed for parameters estimation: using MATLAB, the system of difference equations is solved numerically, with initial chosen values for parameters and state variables from Table 1. Model out-comes are compared with the field data, and the Levenberg–Marquardt optimization algorithm determines a new set of parameters' values with the model outcomes in a better fit to the field data [21, 22]. After new parameters' values are deter- mined by this optimizer, the system of difference equations is solved numerically using these new parameters' value, and the model outcomes are compared again with the field data. This iteration process between parameter updating and numerical solutions of the system of difference equations continue till convergence criteria for the parameters are met. In this pro- cess of estimating, parameters about one thousand values are chosen using a random process for each of the parameters to be estimated.

We can also notice that the number of individuals who have recovered and died continues to rise slightly compared to the number of infections that amounted to about 1,000 on April 05. Based on the estimated parameters, we simulatethe peak or not. It can be seen from Figure 2 that by May 07, the number of active cases in India will exceed 3000 individuals, more than 600 recovered individuals, and about 800 deaths. This simulation shows the efficiency of the straegy of control used by the Moroccan authorities, by slowing the spread of the epidemic compared to other affected countries such as Italy, Spain, and the United States that are in the top of the list of affected countries. To justify the increase of the number of infections in India despite the use of the national quarantine, we estimate the number of people who do not respect and those respecting the quarantine in Figure 3, where we can see that the number of totally con- trolled individuals starts increasing slowly from the beginning of the closure on March 20 to reach about 30 million at the beginning of May. But the number of partially controlled individuals rises continuously to reach about 15 million individuals before starting decreasing towards 5 mil- lion by the first week of May. That means that the population at risk in India in the first month of the national closure is more than ten million. This result explains the growing number of infections and deaths in the country despite the nationwide quarantine.

Conclusion

We determine numerically the residuals of the parameters' estimation, and then, we examine the accuracy of the normality of the estimation of the parameters. In order to carry out this examination, we generate the residuals of the three functions of infected, removed, and dead populations, depicted in Figure 4, where it can be seen that there are small residuals compared to the values of each population. While Figure 5 displays the corresponding histograms of Ik–Idatain (a) and Rk − Rdata in (b) andDk − Ddata in (c) for the models (1), (2), (3), (4), (5) and (6) from March 19 toMay 07 to predict the course of the events and to see if the status of the COVID-19 infections in India.

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