STUDY OF QUEUEING DYNAMICAL MODEL WITH IMPATITENT BEHAVIOUR OF CUSTOMERS

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ABSTRACT

In this paper we will study about the cost effectiveness of M/M/1/N Queue system with impatient nature of customers**.** The arriving customers have been expected to balk with some probability and the rate of reneging is spread exponentially. With the assistance of Markov process techniques, the steady state probability equations have been developed. A Solution for the matrix structure has been derived to define the different output of the measure. To find the optional service rate, a mathematical modeling has been developed and the various Parametric values between the degree of waiting time and the total expected cost have been established.

Keyword:- Balking, Reneging, Cost analysis, Exponential distribution, steady state probability.

1 INTRODUCTION

The earlier researchers did not count the rate of impatient customer in the study of queue system but as the time goes on civilization progress the population density not only of human beings but also vehicles wore increase, the impatient behavior of customers rate become significant. It could not be ignored in situations arising hospital emergency room, toll booth, inventory control system storing valuable goods etc. Practically it has been observed that the cost factor play vital role to survey & study a queue system, for which equilibrium level queue decision model is required. It is because the two conflicting cost offering the service i.e. the cost of providing service with waiting time cost and the cost of preventing the queue computation of total expected operational cost per unit time for the system is required.

Haight (1957,1959) made the first attempt to discuss the effect of balking and reneging phenomenon in queueing problem was further studied by Ancker & Gafarin (1963) and Abou & Harire (1992), Singh TP (1985) incorporated the reneging concept (reluctant a customer remains in line after joining the queue & waiting) in serial queue network and obtained transient solution. Ashok & Taneja (1983) studied the cost analysis of multichannel queue system where in both the arrival & service intensities are subject to alterations. Man Singh & Umed Singh (1994) studied the steady state behavior for impatient customers. Neetu Gupta

(2009) etal made and effort of the bulking and reneging effect on the performance of a queue system under certain parametric constraints. Recently, Singh T.P. & Arti (2014) discussed the cost analysis of a queue system with impatient customers.

This study is further an extended work of above said authors and explores the cost analysis of queue dynamical system in a wider sense. The degree of waiting time equilibrium phenomenon has been derived. The concept has been made clear through numerically illustration along with graphically presentation, making analysis more meaning full and relevant.

2 MODEL FORMULATION

The Mathematical modeling for the stated system under study can be depicted as below

2.1 ASSUMPTIONS

1. Customers arrive at the system one by one in a poisson fashion with expected arrival rate λ. On arrival a customer either decides to join the queue with probability b_n or balk from the system on observing the long queue with probability $1-b_n$ when n customers are ahead (n=1,2,3,4, N-1) where N is maximum number of customers in the system i.e.

   $0 \leq b_{n-1} \leq b_n < 1 \qquad 1 \leq n \leq N-1$

   $bn = 0 \qquad n \geq N$

2. On joining queue each customer has to wait a certain time period T in order to start the service. In case customers has not received service at once he feels irritated, after a while gets impatient and reneges from queue without being served. Here, the time T is a random variable following exponential distribution and its probability distribution function is given by $f(t) = \alpha e^{-\alpha t}$ $t \geq 0, \alpha \geq 0$

   Where α is the degree of waiting rate. Since the arrival and departure of impatient customers without service are independent, the function of customer's average reneging rate r_n is directly proportional to degree of waiting rate α mathematically given by

   $r(n) = (n - i)\alpha \qquad i \leq n \leq N$

   $= 0 \qquad n > N \qquad i = 0,1,2,3\ldots\ldots\ldots$

3. Queue discipline is FIFO, Once the service starts, it precedes till its completion.
4. The service time has been assumed to be exponentially distributed whose probability distribution function is given by

   $G(t) = \mu e^{-\mu t}, \qquad t \geq 0, \mu > 0$ where μ is service rate

3. FORMATION OF DIFFERENTIAL DIFFERENCE EQUATION

Define, P_n = Probability that there are n customers in the system,

b_n = Probability that on arrival a customer decides to join the queue or bulk with the probability $1-b_n$. The steady state probability differential difference equation governing the model can expressed as

$\lambda\, b_{n-1}\, P(n-1) + (\mu + n\alpha)\, P\,(n+1) = [\lambda\, b_n + \mu + (n-1)\alpha]\, P(n).$ For n = 1,2,3 ……..N-1

$\lambda\, b_{N-1}\, P(N-1) + [\mu + (N+1)\,\alpha]\, P(n),$ For n = N

$\mu\, p(1) = \lambda\, p(0)$ For n = 0

Let us discuss the cost model for N = 5

The following equation are obtained

$\mu\, p(1) = \lambda\, p(0)$ For n = 0 (1)

$\lambda\, P(0) + (\mu + \alpha)\, P\,(2) = [\lambda\, b1 + \mu]\, P(1)$ For n = 1 (2)

$\lambda\, b_1 P(1) + (\mu + 2\alpha)\, P\,(3) = [\lambda\, b_2 + \mu + \alpha]\, P(2)$ For n = 2 (3)

$\lambda\, b_2 P(2) + (\mu + 3\alpha)\, P\,(4) = [\lambda\, b_3 + \mu + 2\alpha]\, P(3)$ For n = 3 (4)

$\lambda\, b_3 P(3) + (\mu + 4\alpha)\, P\,(5) = [\lambda\, b_4 + \mu + 3\alpha]\, P(4)$ For n = 4 (5)

$\lambda\, b_4 P(4) = [\mu + 4\alpha]\, P(5)$ (6)

Solution Methodology

Writing the above equations in matrix form

$$\begin{vmatrix} -\lambda & \mu & 0 & 0 & 0 & 0 \\ \lambda & -(\lambda b_1 + \mu) & (\mu + \alpha) & 0 & 0 & 0 \\ 0 & \lambda b_1 & -(\lambda b_2 + \mu + \alpha) & (\mu + 2\alpha) & 0 & 0 \\ 0 & 0 & \lambda b_2 & -(\lambda b_3 + \mu + 2\alpha) & (\mu + 3\alpha) & 0 \\ 0 & 0 & 0 & \lambda b_3 & -(\lambda b_4 + \mu + 3\alpha) & (\mu + 4\alpha) \\ 0 & 0 & 0 & 0 & \lambda b_3 & -(\mu + 4\alpha) \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{vmatrix}$$

On solving by usual method, we get

$P(0) = K$

$P(1) = \frac{\lambda}{\mu} K$

$$P(2) = \lambda^{2} \frac{b_1 K}{\mu(\mu + \alpha)}$$

$$P(3) = \lambda^{3} \frac{b_1 b_2 K}{\mu(\mu + \alpha)(\mu + 2\alpha)}$$

$$P(4) = \lambda^{4} \frac{b_1 b_2 b_3 K}{\mu(\mu + \alpha)(\mu + 2\alpha)(\mu + 3\alpha)}$$

$$P(5) = \lambda^{5} \frac{b_1 b_2 b_3 b_4 K}{\mu(\mu + \alpha)(\mu + 2\alpha)(\mu + 3\alpha)(\mu + 4\alpha)}$$

Applying initial conditions

$$\sum_{i=1}^{5} P_i = 1$$

i.e. P(0) + P(1) + P(2) + P(3) + P(4) + P(5) =1

Putting the values we get

$$K = \left[1 + \frac{\lambda}{\mu} + \lambda^2 \frac{b_1}{\mu(\mu+\alpha)} + \lambda^3 \frac{b_1 b_2}{\mu(\mu+\alpha)(\mu+2\alpha)} + \lambda^4 \frac{b_1 b_2 b_3}{\mu(\mu+\alpha)(\mu+2\alpha)(\mu+3\alpha)} + \lambda^5 \frac{b_1 b_2 b_3 b_4}{\mu(\mu+\alpha)(\mu+2\alpha)(\mu+3\alpha)(\mu+4\alpha)}\right]^{-1}$$

Cost analysis and performance measure

Expected number of cost in Queue i.e. waiting customer in Queue and system

Balking Rate (BR) = $\sum_{n=1}^{N} \lambda(1 - b_n) P(n)$

Reneging Rate (R.R.) = $\sum_{n=1}^{N} (n-1)\alpha\, P(n)$

L.R. = B.R. + R.R.

Where L.R is cost incurred due to customers loss.

μ = Control variable

Our objective is to control the service rate to minimize system's total average cost per unit.

C_1 = Cost per unit time when server is busy.

C_2 = Cost per unit time when customer join in the queue and waits for service.

C_3 = Cost per unit time when a customer balks or reneges.

F(μ*) = Expected functional cost of the system per unit time.

Total expected functional cost of the system per unit time

$T\,F(\mu^*) = C_1 P(B) + C_2\,E(NL) + C_3\,L.R$

P(B) = busy probability of server

NUMERICAL ILLUSTRATION

Consider maximum number of customer in system N = 5

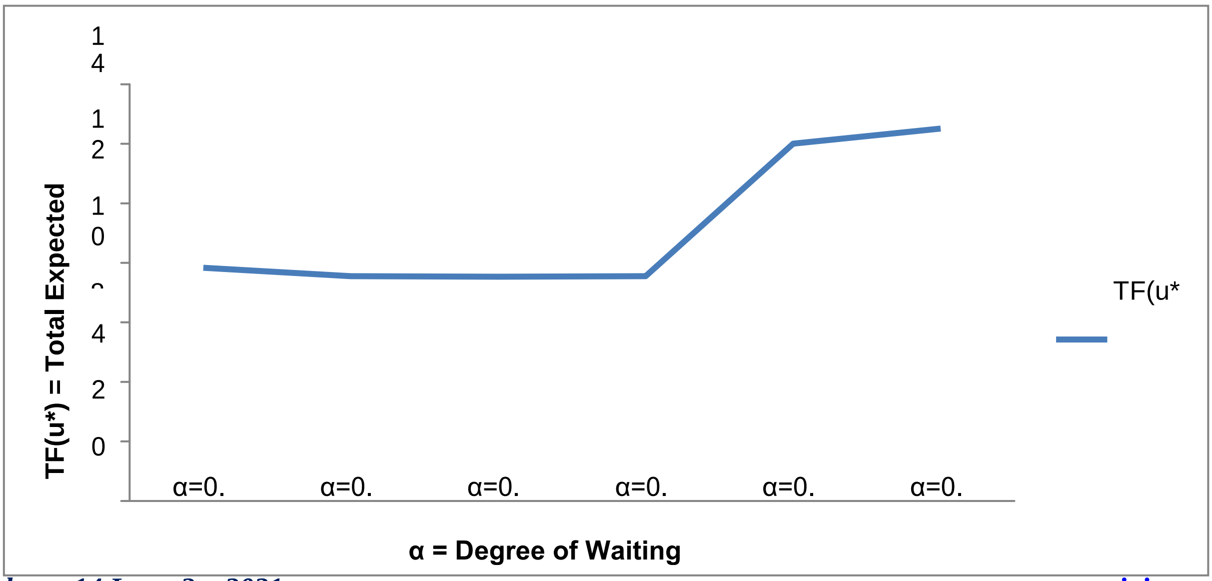
The probability **bn**= $\frac{1}{n+1}$ & cost element $\mathbf{C_1}$ = 10, $\mathbf{C_2}$ = 8, $\mathbf{C_3}$ = 16

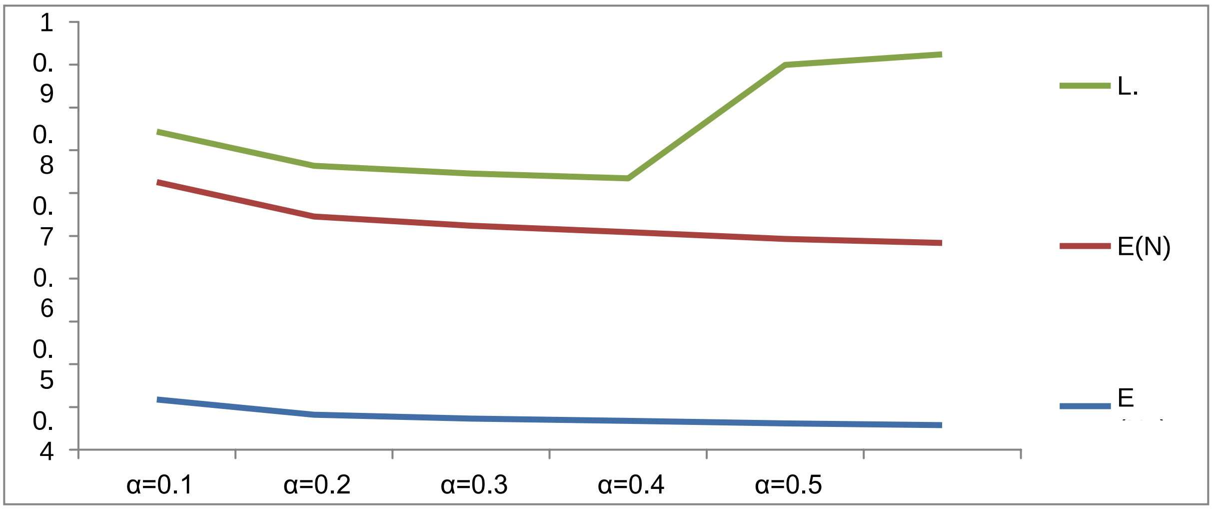
Table – 1 for α = 0.1

Λ ⟶	**.4**	**.5**	**.6**	**.7**	**.8**
E(Nq)	0.061055	0.117363	0.1286619	0.16658565	0.210273
E(N)	0.386512	0.508244	0.5739055	0.659896	0.74826574
L.R.	0.0751841	0.1180925	0.1572375	0.20920	0.261520409
T F(μ*)	5.690945	7.82838	9.54515	11.6434	13.8665

Table – 2 for α = 0.5

Λ ⟶	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**
E(Nq)	0.117363	0.081815	0.72591	0.067199	0.061837	0.057326
E(N)	0.508244	0.463636	0.451379	0.44144	0.431094	0.426142
L.R.	0.1180925	0.1182845	0.1220313	0.12579	0.406961	0.440762
T F(μ*)	7.82834	7.547056	7.53322	7.55023	12.0060	12.51086





ANALYSIS OF GRAPH AND TABLE

1. We select the fix degree of waiting time rate α = 0.1 & change value of arrival rate of customers λ, result has been summarized in Table – 1.

   Table 1 show that as the value of λ increase then expected number of customer in queue and expected number of customer in system increase and mean rate all increase the total expected functional cost increase with increase of λ. And average rate of customer loss all increase with increase with increase of λ.

   While the graph of Table – 2 shown that an increasing value of α Expected number of customer in system as well as expected number of customer in queue and mean rate all decreases. A is the degree of waiting rate T. Since the Reneging rate is the function of α. As the value degree of waiting time rate increase we find total expected functional cost in the beginning slowly decrease. But when value of α increase at 0.4 their sudden Jump in the total expected cost. We say at α = 0.4 is point of equilibrium and where the degree of waiting time rate and total expected cost balance each other. It is because of reneging and balking rate increase fastly after this stage there will be loss of system more customer will go.

CONCLUSION

We have discussed queue system with impatient customers and developed the steady state probability equations. The Matrix form of the solution has been derive. We formulate a cost model to determine the optional service rate and total expected cost of the system per unit time. Although this function is too complicated to derived the explicit expression for optimal service rate, even than we have made an attempt to evaluate numerically to performance measures & the optimal service rate for the system.

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