Study of a Bulk Queueing Model with Working Vacation

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ABSTRACT: *In this paper, attempt to study a markovian queueing model with single server working vacation in which customer's arrive according to Poisson process with parameter λ. The service is providing to the customer with two different service rate which depends upon batch size(γ, δ) which is the maximum 'γ' and minimum number 'δ' of customers in the system. If immediately after providing the service if the numbers of the customers are less than 'γ' then server will on working vacation and serves with rate μ_1. If queue size is greater than 'γ' then server is busy and serves the customer with rate μ_2. Here time dependent solution of mean number of arrivals, expected queue length on working vacation and queue length on busy period are derive. This paper concludes with graph which shows the relation between queue length and service rate.*

KEYWORDS: - Queueing Model, bulk arrival, bulk service, queue length, busy period.

INTRODUCTION: - It is a very simple and elegant method is used to studying a bulk queueing model with working vacation. The queueing model with batch service has an extensive literature, dating back to bailey [1], Boxma [2] worked on waiting time and vacation. Champernowne[3] has given a solution on queueing model and constant parameter with single server. Senthalinathan [4, 6] studied about multiple working vacations when there are bulk arrival of customer and also servive providing to them in bulk and then analyses queueing model with batch size depent on service and working vacation. Nagarajan [7] worked on batch service queue with unreliable server having compulsory vacation impose on them. Here we are considering a more general case, namely, the size(γ, δ), which takes a minimum of 'γ' assuming that, arrival of the customer takes place by following poisson process with rate λ, and service times of batches depends on the size of batch. In this model, the arrivals process is assumed to follow Poisson with parameter λ and the service times of batches depend upon the batch size.

The service time is exponentially distributed which depends upon batch size(γ, δ) by a single server. If the queue is empty then server goes on vacation, when the queue size is less then 'γ' the server is on working vacation & if the queue size is greater than 'γ' then server is busy. As an example, consider a taxi car giving services between two cities. The driver starts the trip if he gets at least 'γ' customers, if there are more than 'δ' customers the driver takes only 'δ' customers in the car in a trip. After the trip, if he finds that the number of customers waiting for taxi is less than 'γ' than he goes for the working vacation. After working vacation, the drivers returns and if the number of customers waiting for taxi is still less than 'γ' he remains idle until at least 'γ' customers are available.

ANALYSIS OF MODEL

$P_{i,n}(t) = P\{x(t) = i,\ y(t) = n\}$

$x(t) = 0 \qquad y(t) = 0 \qquad$ vacation

$x(t) < \gamma y(t) = 1 \qquad$ working vacation

$x(t) > \gamma y(t) = 2 \qquad$ busy

The difference-differential equations governing the model are

$$P'_{00}(t) = -\lambda P_{00}(t) + \mu_1 P_{0,1}(t) + \mu_2 P_{0,2}(t) \qquad \dots (1)$$

$$P'_{0,1}(t) = -(\lambda+\mu)P_{0,1}(t) + \lambda P_{00}(t) + \mu_1 P_{11}(t) + \mu_2 P_{12}(t) \qquad \dots (2)$$

$$P'_{n,1}(t)) = -(\lambda+\mu_1)P_{n,1}(t) + \lambda P_{n-1}(t) + \mu_1 P_{n+1,11}(t) \quad 1 \le n \le \gamma - 1 \qquad \dots (3)$$

$$P'_{0,2}(t) = -(\lambda + \mu_2) P_{0,2}(t) + \lambda P_{\gamma-1,1}(t) + \mu_2 P_{n,2}(t) \ n > \gamma \qquad \dots (4)$$

$$P'_{n,2} = -(\lambda + \mu_2) P_{n,2}(t) + \lambda P_{n-1,2}(t) + \mu_2 P_{n+\gamma,2}(t) \ n \ge 1 \qquad \dots (5)$$

Taking Laplace Transformation of equation (1) to (5)

$$S\overline{P}_{00}(s) - P_{00}(0) = -\lambda \overline{P}_{0,0}(s) + \mu_1 \overline{P}_{0,1}(s) + \mu_2 \overline{P}_{0,2}(s)$$

$$(S+\lambda)\overline{P}_{00}(s) - 1 = \mu_1 \overline{P}_{01}(s) + \mu_2 \overline{P}_{0,2}(s)$$

$$P_{00}(0) = 1 \qquad \dots (6)$$

$$\overline{P}_{00}(s) = \frac{1}{S+\lambda}(\mu_1 \overline{P}_{01}(s) + \mu_2 \overline{P}_{02}(s) + 1)$$

$$S\,\overline{P}_{01}(s) - P_{01}(0) = -(\lambda+\mu)\ \overline{P}_{01}(s) + \lambda\overline{P}_{00}(s) + \mu_1\overline{P}_{11}(s) + \mu_2\overline{P}_{12}(s)$$

$$\Rightarrow \quad (S+\lambda+\mu)\ \overline{P}_{01}(s) = \lambda\overline{P}_{00}(s) + \mu_1\overline{P}_{11}(s) + \mu_2\overline{P}_{12}(s) \qquad \ldots(7)$$

$$\overline{P}_{01}(s) = \frac{1}{S+\lambda+\mu}(\lambda\overline{P}_{00}(s) + \mu_1\overline{P}_{11}(s) + \mu_2\overline{P}_{12}(s)$$

$$S\overline{P}_{n,1}(s) - P_{n,1}(0) = -(\lambda+\mu_1)\overline{P}_{n,1}(s) + \lambda\overline{P}_{n-1,1}(s) + \mu_1\overline{P}_{n+1,1}(s) + \mu_2\ \overline{P}_{n+1,2}(s)$$

$$(S+\lambda+\mu_1)\ \overline{P}_{n1}(s) = \lambda\overline{P}_{n-1,1}(s) + \mu_1\overline{P}_{n+1,1}(s) + \mu_2\overline{P}_{n+1,2}(s) \qquad \ldots(8)$$

$$\overline{P}_{n,1}(s) = \frac{1}{S+\lambda+\mu}(\lambda\overline{P}_{n-1,1}(s) + \mu_1\overline{P}_{n+1,1}(s) + \mu_2\overline{P}_{n+1,2}(s))$$

$$\overline{P}_{0,2}(s) = \frac{1}{(S+\lambda+\mu_2)}(\lambda\overline{P}_{a-1,1}(s) + \mu_2\overline{P}_{n,2}(s)) \qquad \ldots(9)$$

$$\overline{P}_{n,2}(s) = \frac{1}{(S+\lambda+\mu_2)}[\lambda\overline{P}_{n-1,2}(s) + \mu_2\overline{P}_{n+a,2}(s)] \qquad \ldots(10)$$

$\Rightarrow$ The characteristic equation from equation (10), assuming h(z) = 0

$$(S+\lambda+\mu_1)\ \overline{P}_{n1}(s) = \lambda\overline{P}_{n-1,1}(s) + \mu\overline{P}_{n+1,2}(s)$$

$$\Rightarrow \quad \mu_2 z^2 - (S+\lambda+\mu_1)z + \lambda = 0 \qquad \ldots(11)$$

From (10)

$$\Rightarrow \quad \mu z^{a+1} - (S+\lambda+\mu_2)\, z + \lambda = 0 \qquad \ldots(12)$$

Suppose that $f(z) = -(S+\lambda+\mu_2)\, z$ and $g(z) = \mu z^{\gamma+1} + \lambda$

Consider the circle |z| = 1–€ where δ is arbitrarily small & $z = (1-€)e^{i\theta}$, it can be shown that on the contour of the circle |g(z)| < |f(z)|. Hence by Rouche's Theorem

f(z) and f(z) + g(z) will have the same number of zeros inside (z) = 1–€. Since f(z) has only one zero inside the circle f(z) + g(z) = h(z) will also have only one zero inside (z) = 1–€, this root of h(z) = 0 is real and unique

$$\text{iff } \rho = \frac{\lambda}{a\mu_1} < 1 \ \ \& \ (0 < \alpha < 1)$$

And other roots $\beta_i \geq 1$

Then α satisfies the equation

$$a\alpha = \frac{\lambda}{\mu_1} = \frac{\alpha(1-\alpha^a)}{1-\alpha} = \alpha + \alpha^q + \ldots + \alpha^a$$

And $$\frac{\lambda}{\mu_2} = \frac{\alpha(1-\alpha^a)}{1-\alpha}$$

From equation (11)

$$z = \frac{(S+\lambda+\mu) \pm \sqrt{(S+\lambda+\mu_1)^2 - 4\mu_2\lambda}}{2\mu_2}$$

Let

$$\alpha = \frac{(S+\lambda+\mu) + \sqrt{(S+\lambda+\mu_1)^2 - 4\mu_2}}{2\mu_2}$$

$$\beta = \frac{(S+\lambda+\mu) - \sqrt{(S+\lambda+\mu_1)^2 - 4\mu_2}}{2\mu_2}$$

Hence by Rouche's theorem

$$\overline{P}_{n,1}(s) = \overline{P}_{0,1}(s)\alpha - \overline{P}_{0,2}(s)\mu_2 R^n$$

$$\overline{P}_{i-1,1}(s) = \overline{P}_{0,1}(s)\alpha\frac{\lambda}{S+\lambda+\mu_1} - \overline{P}_{0,2}(s)\mu_2\frac{\lambda}{S+\lambda+\mu_1}R^{a-1}$$

$$\overline{P}_{0,2}(s) = \frac{\overline{P}_{01}(s)\alpha\left[\left(\frac{\lambda}{S+\lambda+\mu_1}\right)\left(\frac{\lambda}{S+\lambda+\mu_2}\right)\alpha\right]}{\left[1+\mu_2\left(\frac{\lambda}{S+\lambda+\mu_1}\right)\left(\frac{\lambda}{S+\lambda+\mu_2}\right)\alpha R^n\right]}$$

Let $$D_1 = \frac{\lambda^2\alpha(S+\lambda+\mu_1)(S+\lambda+\mu_2)}{(S+\lambda+\mu_1)(S+\lambda+\mu_2)[(S+\lambda+\mu_1)(S+\lambda+\mu_2)+\mu_2\lambda^2\alpha R^n]}$$

$$D_1 = \frac{\lambda^2\alpha}{(S+\lambda+\mu_1)(S+\lambda+\mu_2)+\mu_2\lambda^2\alpha R^n}$$

From equation (6)

$$\overline{P}_{00}(s) = \frac{\overline{P}_{01}(s)}{S+\lambda}(\mu_1+\mu_2 D_1) + \frac{1}{S+\lambda}$$

$$\overline{P}_{n,1}(s) = \overline{P}_{01}(s)[\alpha - D\mu_2\, R^{n+1}]$$

$$\overline{P}_{n,2}(s) = \overline{P}_{01}(s)\frac{R^n\lambda^2\alpha}{(S+\lambda+\mu_1)(S+\lambda+\mu_2)+\mu_2\lambda^2\alpha R^n}$$

According to normalizing condition

$$\sum_{i=0}^{n}\overline{P}_{i0}(s)+\overline{P}_{i,1}(s)+\overline{P}_{i2}(s)=\frac{1}{S}$$

Steady state probabilities

$$P_{in} = \lim_{s\to\infty} S\overline{P}_{i,n}(s)$$

$$P_{00} = \lim_{s\to\infty} S\overline{P}_{01}(s).\frac{\mu_1}{S+\lambda}+S\overline{P}_{01}(s)\frac{\mu_2}{S+\lambda}.\frac{\lambda^2\alpha}{(S+\lambda+\mu_1)}(S+\lambda+\mu_2)+\mu_2\lambda^2\alpha R^n$$

$$+\frac{1}{S+\lambda}S$$

$$P_{00} = P_{01}\left[\frac{\mu_1}{\lambda}+\frac{\mu_2}{\lambda}K\right]$$

$$K = \left(\frac{\lambda^2\alpha}{\lambda+\mu_1)(\lambda+\mu_2)+\mu_2\lambda^2\alpha R^n}\right)$$

$$P_{n1} = P_{01}\left(\left(\frac{\lambda}{\mu_1}\right)^n - K\mu_2 R^n\right)$$

$$P_{n2} = \frac{\lambda^2\alpha R^n}{(\lambda+\mu_1)(\lambda+\mu_2)+\mu_2\lambda^2\alpha R^n}.P_{01}, \qquad P_{n2} = P_{01} K R^n$$

EXPECTED QUEUE LENGHT ON VACATION

$$L_{q_v} = \sum_{i=0}^{n} n\, P_{00} = 0$$

$\Rightarrow$ server is on working vacation if there is no customer

$$L_{q_{wv}} = \sum_{n=1}^{a} n\, P_{(n1)}$$

$$L_{qwV} = P_{01}\left\{\sum_{n=1}^{a-1} n\left[\left(\frac{\lambda}{\mu_1}\right)^n - \mu_2 R^n\left(\frac{\lambda^2\alpha}{(\lambda+\mu_1)(\lambda+\mu_2)+\mu_2\lambda^2\alpha R^n}\right)\right]\right\}$$

$$L_{qB} = \sum_{n=a}^{\infty} n\, P_{n2}$$

$$L_{qB} = P_{01}\sum_{n=a}^{\infty} n\frac{\lambda^2\alpha\, R^n}{(\lambda+\mu_1)(\lambda+\mu_2)+\mu_2\lambda^2\alpha R^n}$$

Expected Busy Periods

$$P_{00} = \frac{E(\text{idle Period})}{E(\text{idle Period})+E(\text{Busy Period})}$$

$$E(IdP) = \frac{1}{\lambda}$$

$$P_{00} = \frac{\frac{1}{\lambda}}{\frac{1}{\lambda}+E(BP)} \qquad \Rightarrow P_{00} = \frac{1}{1+\lambda E(BP)}$$

$$\Rightarrow \qquad 1+\lambda\, E_{BP} = \frac{1}{P_{00}} \qquad \Rightarrow E_{BP} = \frac{1-P_{00}}{\lambda P_{00}}$$

The following differential equation

Let $P_{00} = 0.6$, $E_{BP} = \frac{1-0.6}{\lambda(0.6)}$

NUMERICAL ILUSTRATION: Let P00 = 0.06,

$\lambda = 0.1$, $\mu_1 = 0.3$, $\mu_2 = 0.4$

$\lambda = 0.2$, $\mu_1 = 0.5$, $\mu_2 = 0.6$

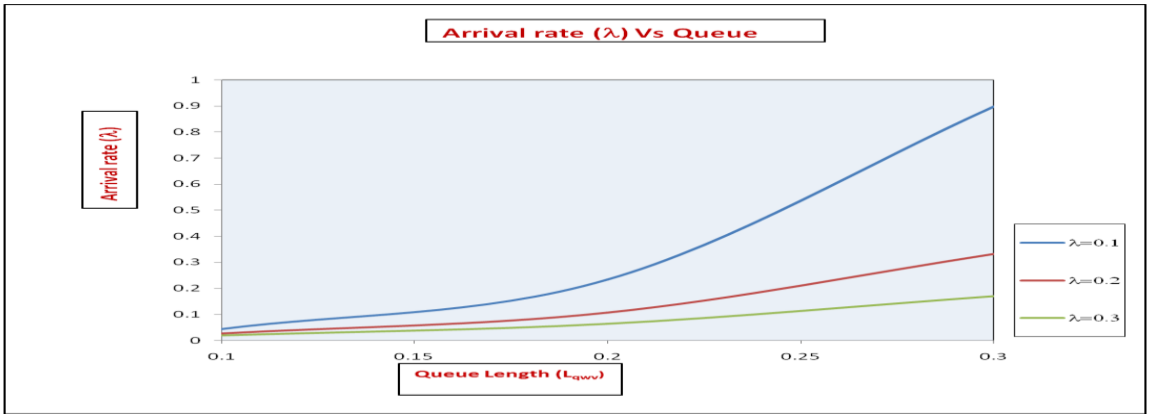
$\lambda = 0.3$, $\mu_1 = 0.7$, $\mu_2 = 0.8$

Table -I

P_{00}	λ	μ_1	μ_2	A	L_{qwv}
0.06	0.1	0.3	0.4	0.0625	0.02648157
0.06	0.2	0.5	0.6	0.05	0.018678107

0.06	0.3	0.7	0.8	0.035714286	0.011639526

Graph-I



We have taken different value of λ, μ_{1}, μ_2 in table –I which shows that as the arrival of the customer in the system increases queue length also increases in the system.

CONCLUSION:- We have discussed queue system with bulk arrival of customers with working vacation and developed the steady state probability equations. The Steady state solution has been derive.. Although this function is too complicated to derived the explicit expression for optimal service rate, even than we have tried an attempt to study system numerically and graphically. Also we have find expected busy period of the system.

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