A COMPARATIVE STUDY OF REDUCED ORDER MODELLING USING DIFFERENTIATION

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ABSTRACT

The higher order systems (HOS) are generally costly and tedious for the analysis purpose. Also, the exact analysis of HOS is very much complex for computation and synthesis. Hence, certain simplification procedures based on practical model consideration or using the mathematical computation techniques are widely used to get lower order systems (LOS) in place of original HOS. The analysis of LOS is easier and faster. The present work deals with a simple mathematical technique known as differentiation in order to obtain LOS. Results obtained from differentiation have been compared with original HOS and also with the other lower order systems already available in the literature. Time response specifications such as: rise time, settling time, peak and peak time have also been compared for HOS & LOS. A comparative study of ISE has also been carried out in the present work for HOS and LOS.

Keywords: Differentiation, HOS, ISE, LOS, SISO.

I. INTRODUCTION

Nowadays, all industries are becoming much complex to perform tasks because of increment in consumers and technologies. Power and control industries have also become the necessities in order to build a system which gives the most economical and appropriate results with least amount of cost. The higher order systems are generally very common in these industries which are much complicated, time consuming for analyst and costly for the synthesis.

Hence, simplification procedures based on either biological approaches or by using any mathematical algorithms are required to reduce a HOS into its equivalent LOS without excluding original HOS's characteristics in reduced LOS. Such techniques for obtaining lower order models from original HOS are known as Model order reduction (MOR) techniques [1-5]. There are various MOR techniques available in the literature to reduce any higher order system into its lower order system [6-9]. In the present work, an algorithm based on differentiation has been used to simplify the HOS. The results obtained from differentiation are comparable in performance with the other existing methods already available in the literature. The method gives better results in comparison to other existing methods, as shown in numerical examples. It has been observed that the stability of original HOS is preserved in its LOS.

The paper is organized as follows: In section 2, the problem formulation is given. The technique used for order reduction in present work, i.e. differentiation has been explained in section 3. Two numerical examples elaborating the suggested differentiation technique have been given in section 4. Further, the conclusions obtained from the present carried out comparative study have been given in section 5 followed by the references.

II. STATEMENT OF PROBLEM

Consider the following SISO HOS of order 'n':

$$G_n(s) = \frac{N(s)}{D(s)} = \frac{a_1 s^{n-1} + a_2 s^{n-2} + ... + a_n}{s^n + b_1 s^{n-1} + b_2 s^{n-2} + ... + b_n} \quad (1)$$

Where, a_i and b_i are constants for i = 1, 2… n.

The important requirement of any reduced order model is that it should contain all the basic important features of the original HOS [9]. If '*r*' represents the order of LOS which is lesser than '*n*', then, the reduced LOS of the system in (1) can be given as:

$$G_r(s) = \frac{N_r(s)}{D_r(s)} = \frac{c_1 s^{r-1} + c_2 s^{r-2} + ... + c_r}{s^r + d_1 s^{r-1} + d_2 s^{r-2} + ... + d_r} \quad (2)$$

Where, c_r and d_r are unknown coefficients which have to be obtained by suggested differentiation technique.

III. DIFFERENTIATION METHOD

The present work is based on simple derivative of polynomials, known as differentiation technique [1]. Both the numerator and denominator of original HOS are first reciprocated and then differentiated suitably many times till the desired order is obtained. Reduced order transfer functions are again reciprocated and normalized in order to match the steady state value.

In order to get the reduced LOS, the steps of differentiation are:

- Initially, the reciprocal of HOS is obtained.
- This reciprocated HOS is differentiated till the desired order is obtained.
- The reduced LOS is then again reciprocated.
- Finally, the normalization is done to match the steady state value of LOS with that of HOS.

IV. NUMERICAL EXAMPLES

In this section, two examples have been given to elaborate the suggested differentiation technique. The performance index ISE known as integral square error in between the transient parts of HOS and LOS has also been calculated to measure the goodness of the LOS (i.e. smaller the ISE, the closer is $G_r(s)$ to $G_n(s)$), which is given by [9]:

$$\textbf{ISE} = \int_0^\infty [\, y(t) - y_r(t)\,]^2 \, dt \quad (3)$$

Where, $y(t)$ and $y_r(t)$ are the unit step responses of HOS and LOS.

4.1 Example-1. Considering a SISO HOS of 8th order given by [2]:

$$G_8(s) = \frac{35 s^7 + 1086 s^6 + 13285 s^5 + 82402 s^4 + 278376 s^3 + 511812 s^2 + 482964 s + 194480}{s^8 + 33 s^7 + 437 s^6 + 3017 s^5 + 11870 s^4 + 27470 s^3 + 37492 s^2 + 28880 s + 9600} \quad (4)$$

4.1.1 ROM using Differentiation Technique

Firstly, reciprocal of (4) is obtained as:

$$G'_8(s) = \frac{35 + 1086s + 13285s^2 + 82402s^3 + 278376s^4 + 511812s^5 + 482964s^6 + 194480s^7}{1 + 33s + 437s^2 + 3017s^3 + 11870s^4 + 27470s^5 + 37492s^6 + 28880s^7 + 9600s^8} \quad (5)$$

Now, the differentiation of (5) is carried out till the desired 2nd order model is obtained. The obtained 2nd order LOS is given in (6):

$$G'_2(s) = \frac{347734080 + 980179200\,s}{26994240 + 145555200\,s + 193536000\,s^2} \quad (6)$$

It is the 2nd order reduced model by differentiation of the reciprocal of the original HOS. Therefore, the (6) is reciprocated again to get the desired 2nd order LOS of the HOS (4) and is given by:

$$G_2(s) = \frac{347734080\,s + 980179200}{26994240\,s^2 + 145555200\,s + 193536000} \quad (7)$$

Now the steady state value of both LOS and HOS is matched; therefore, the steady state correction is compensated by applying the correction factor (k).

The correction factor k = 20.25/5.06 = 4. Therefore, the desired 2*nd* order reduced LOS after correction factor is:

$$G_2(s) = \frac{1390936320\,s + 3920716800}{26994240\,s^2 + 145555200\,s + 193536000} \quad (8)$$

4.1.2 Comparison of Step and Frequency responses

In order to verify the applicability of the LOS from suggested differentiation method and reduced order models already available in the literature for the same HOS by the other researchers, the step and frequency responses have been compared, as shown in Figs. (1-6).

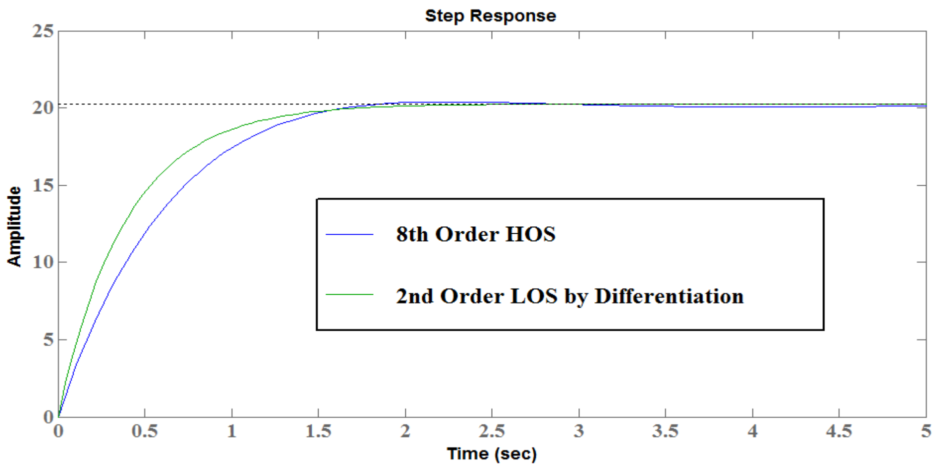

Fig.1. Comparison of Step Responses of HOS (8th order) and LOS (2nd order)

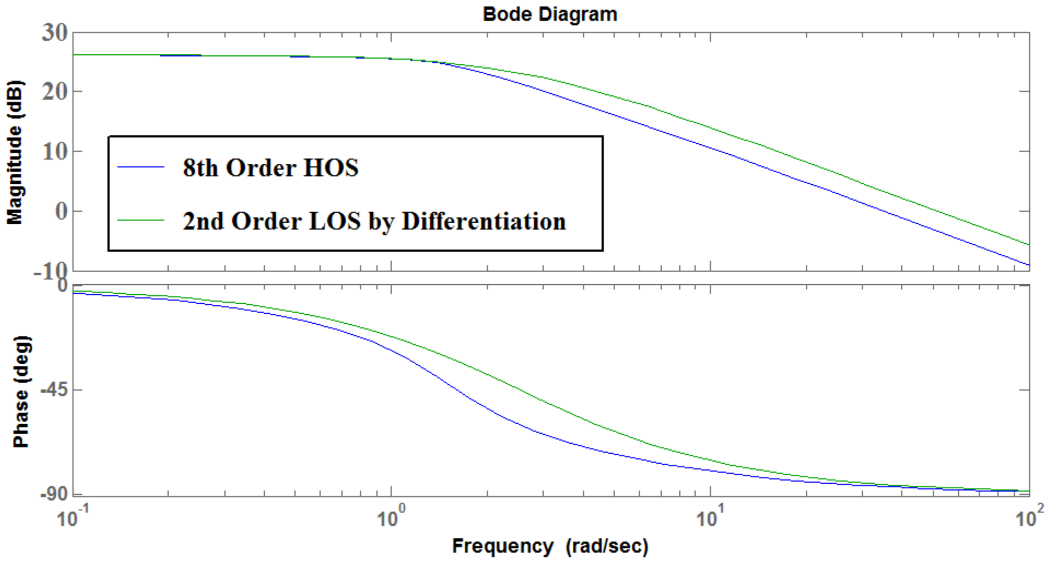

Fig.2. Comparison of Frequency Responses of HOS (8th order) and LOS (2nd order)

4.1.3 Comparison of Differentiation Technique with other Existing Methods [2, 5, 7, 10]

4.1.3.1 2nd order LOS of (4) obtained by Krishnamurthy V. *et al.* [2] is:

$$G_2(s) = \frac{334828.5\, s + 194480}{20123.7\, s^2 + 18116.2\, s + 9600} \quad (9)$$

2nd order LOS of (4) obtained by Satakshi S. *et al.* [7] is:

$$G_2(s) = \frac{944795700.576\, s + 3920724337.6168}{26994240\, s^2 + 145555200\, s + 193536000} \quad (10)$$

Step and frequency responses have been compared as shown in Figs. (3-4).

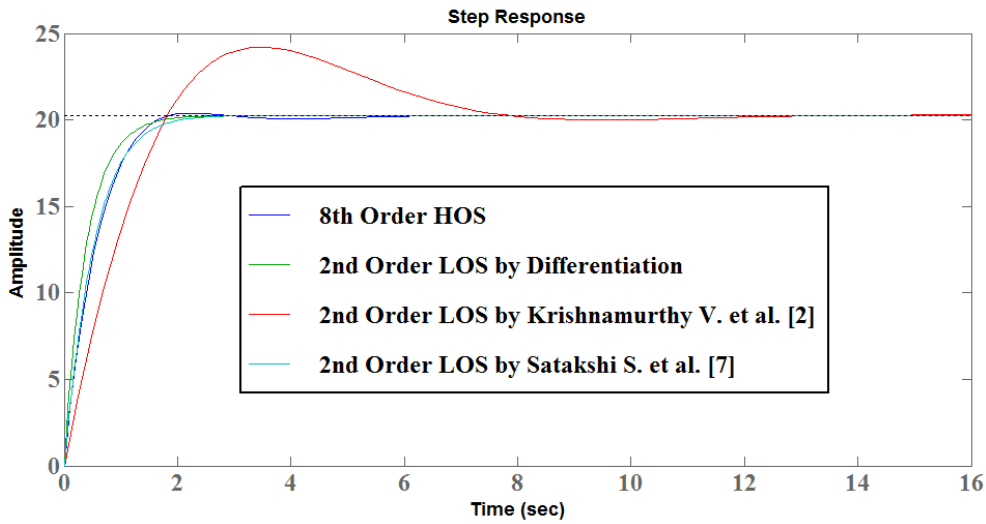

Fig.3. Comparison of Step Responses.

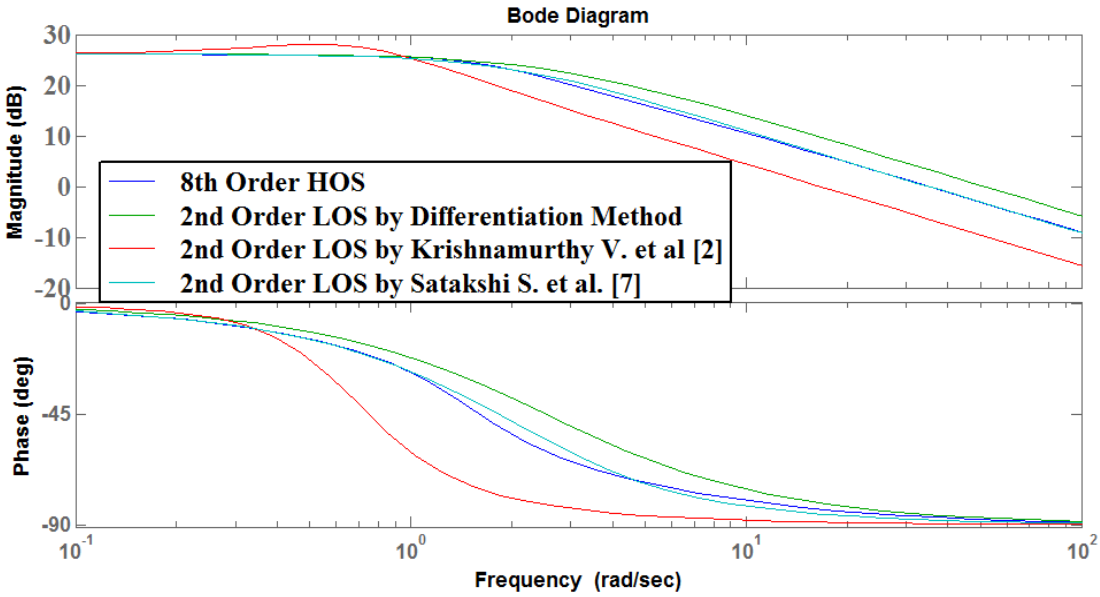

Fig.4. Comparison of Frequency Responses.

4.1.3.2 2nd order LOS of (4) obtained by Sambariya D.K. *et al.* [5] is:

$$G_2(s) = \frac{482964\, s + 194480}{34194\, s^2 + 28880\, s + 9600} \tag{11}$$

2nd order LOS of (4) obtained by Prasad R. *et al.* [10] is:

$$G_2(s) = \frac{17.03\, s + 6.8646}{s^2 + 1.02\, s + 0.3366} \tag{12}$$

Step and frequency responses have been compared in Figs. (5-6).

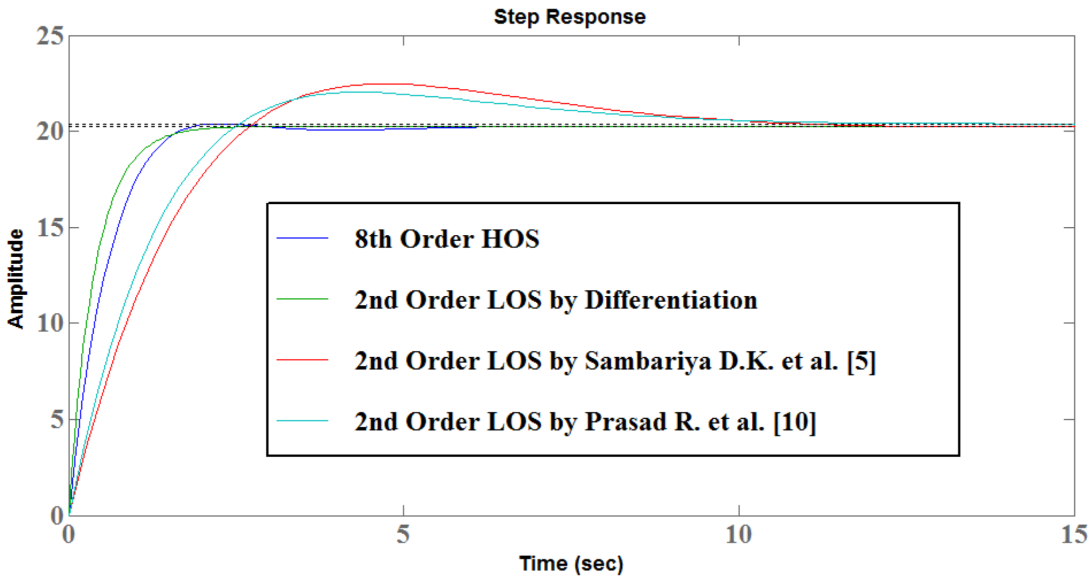

Fig.5. Comparison Of Step Responses.

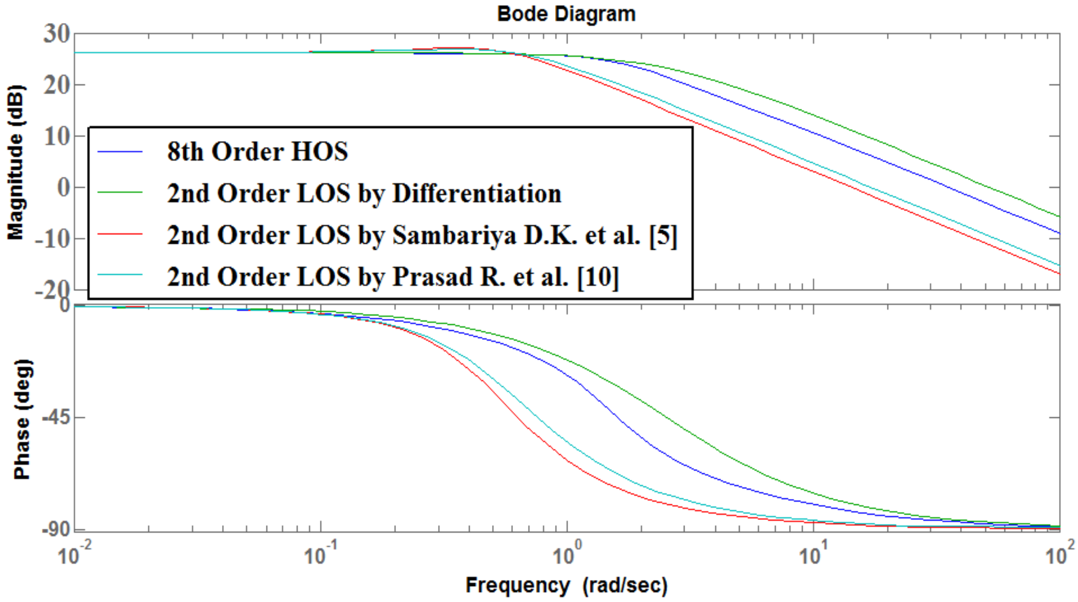

Fig.6.Comparison Of Frequency Responses.

Table I Comparison of Transient Response Parameters

System	Rise Time(sec)	Settling Time(sec)	Peak	Peak Time(sec)
Original HOS (8^{th}Order)	1.07	1.58	20.40	2.23
LOS (2^{nd} Order)	0.88	1.57	20.247	3.07
Krishnamurthy V et al. [2]	1.39	7.05	24.22	3.53
Satakshi S et al. [7]	1.09	1.87	20.249	3.51
Sambariya DK [5]	1.96	9.47	22.50	4.77
Prasad R. et al. [10]	1.80	8.52	22.00	4.36

Table II Comparison of Reduced Models

Method	Reduced Models	ISE
Differentiation Method	$\frac{1390936320\, s + 3920716800}{26994240\, s^2 + 145555200\, s + 193536000}$	4.8328
Krishnamurthy V. *et al.* [2]	$\frac{334828.5 s + 194480}{20123.7 s^2 + 18116.2 s + 9600}$	58.3407
Satakshi S. *et al.* [7]	$\frac{944795700.576 s + 3920724337.6168}{26994240 s^2 + 145555200 s + 193536000}$	0.3278
Sambariya D.K. *et al.* [5]	$\frac{482964 s + 194480}{34194 s^2 + 28880 s + 9600}$	67.0389
Prasad R. *et al.* [10]	$\frac{17.03 s + 6.8646}{s^2 + 1.02 s + 0.3366}$	41.7614

4.2 Example-2. Considering a SISO HOS of 4^{th} order given by [12]:

$$G_4(s) = \frac{4.269 s^3 + 5.10 s^2 + 3.9672 s + 0.9567}{4.39992 s^4 + 9.0635 s^3 + 8.021 s^2 + 5.362 s + 1} \tag{13}$$

4.2.1 ROM using Differentiation Technique

Firstly, reciprocal of (13) is obtained as:

$$G'_4(s) = \frac{4.269 + 5.10 s + 3.9672 s^2 + 0.9567 s^3}{4.39992 + 9.0635 s + 8.021 s^2 + 5.362 s^3 + s^4} \tag{14}$$

Now, the differentiation of (14) is carried out till the desired 2^{nd} order model is obtained. The obtained 2^{nd} order LOS is given in (15):

$$G'_2(s) = \frac{7.9344 + 5.7402\, s}{16.042 + 32.172\, s + 12\, s^2} \tag{15}$$

It is the 2^{nd} order reduced model by differentiation of the reciprocal of the original HOS. Therefore, the (15) is reciprocated again to get the desired 2^{nd} order LOS of the HOS (13) and is given by:

$$G_2(s) = \frac{7.9344 s + 5.7402}{16.042 s^2 + 32.172\, s + 12} \tag{16}$$

Now the steady state value of both LOS and HOS is matched; therefore, the steady state correction is compensated by applying the correction factor (k).

The correction factor k = 0.9567/0.4784 = 1.9998. Therefore, the desired 2^{nd} order reduced LOS after correction factor is:

$$G_2(s) = \frac{15.8672\, s + 11.4793}{16.042\, s^2 + 32.172\, s + 12} \tag{17}$$

4.2.2 Comparison of Step and Frequency responses

In order to verify the applicability of the LOS from suggested differentiation method and reduced order models already available in the literature for the same HOS by the other researchers, the step and frequency responses have been compared, as shown in Figs. (7-10).

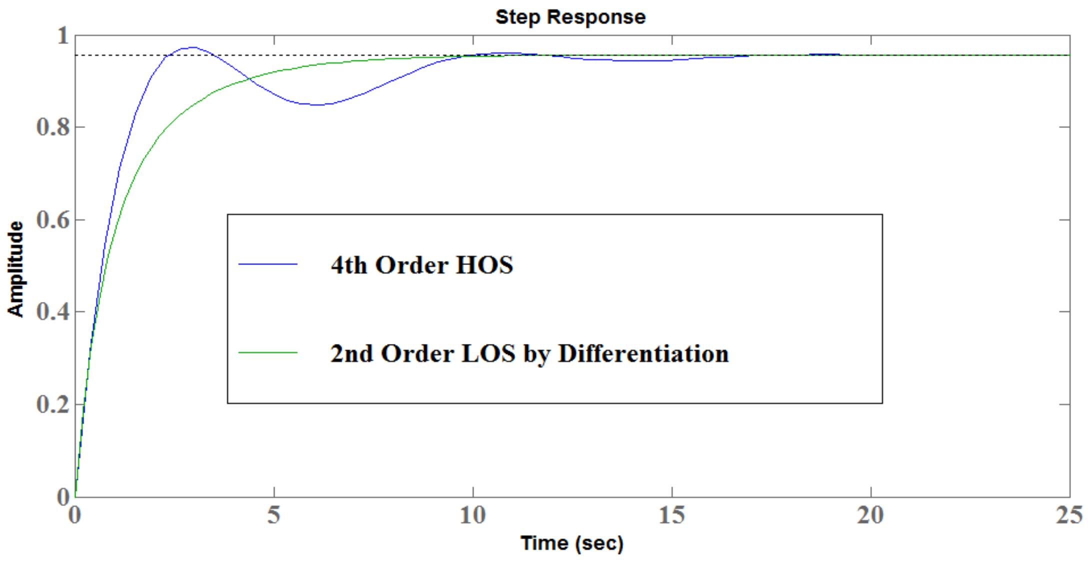

Fig.7. comparison of step responses of HOS (4th order) and LOS (2nd order).

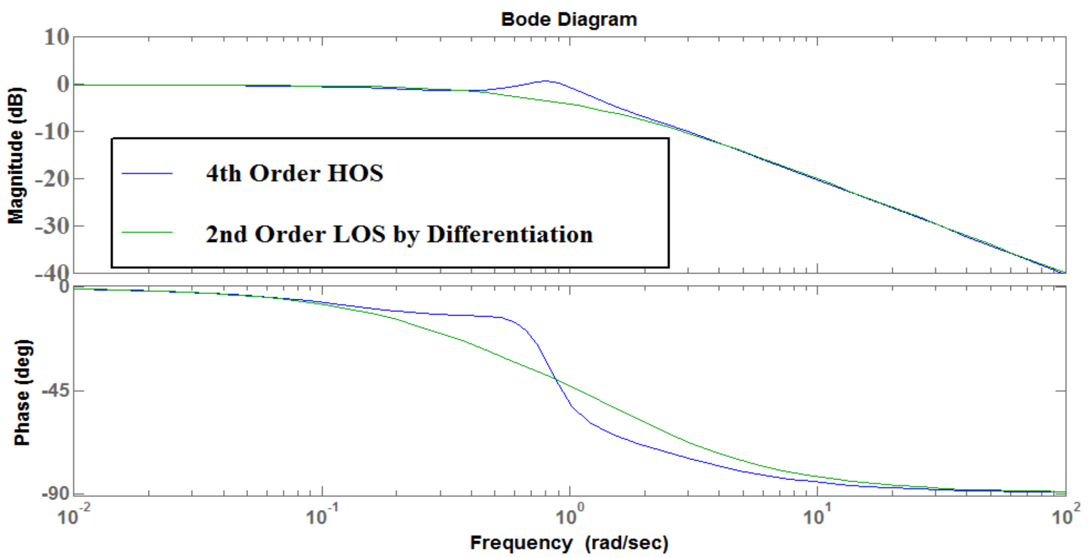

Fig.8. comparison of frequency responses of HOS (4th order) and LOS (2nd order).

4.2.3 Comparison of Differentiation Technique with other Existing Methods [13, 14]

2nd order LOS of (13) obtained by Aguirre L. [13] is:

$$G_2(s) = \frac{0.2211\, s + 0.0702}{s^2 + 1.9531\, s + 0.1415} \tag{18}$$

2nd order LOS of (13) obtained by Bansal J. *et al.* [14] is:

$$G_2(s) = \frac{1.869\, s + 0.5585}{s^2 + 2.663\, s + 0.5838} \tag{19}$$

Step and frequency responses have been compared as shown in Figs. (9-10).

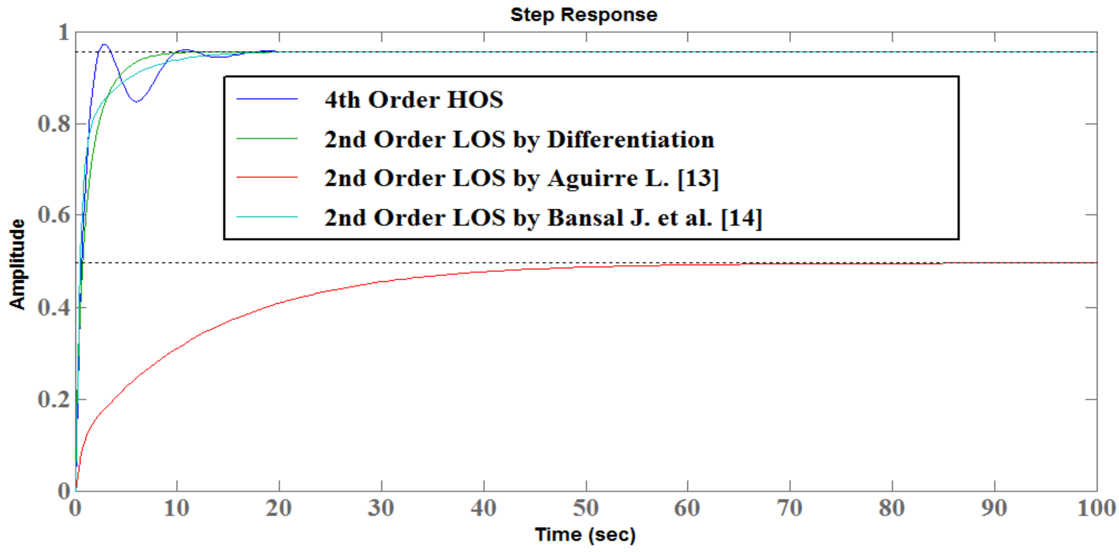

Fig.9. comparison of step responses.

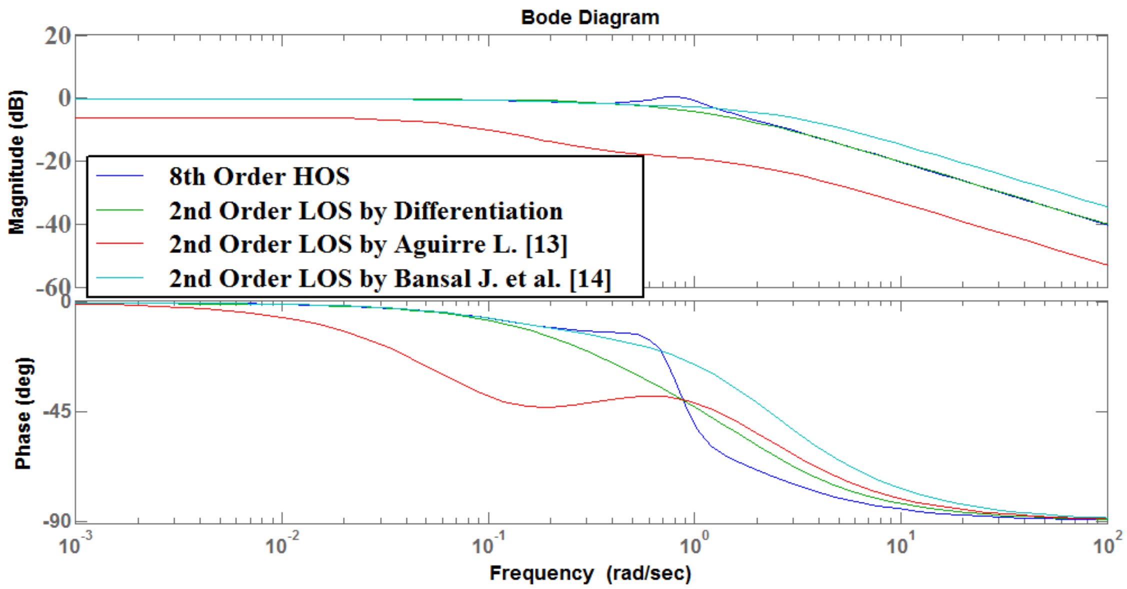

Fig.10. comparison of frequency responses.

Table III Comparison of Transient Response Parameters

System	Rise Time(sec)	Settling Time(sec)	Peak	Peak Time(sec)
Original HOS (4th Order)	1.5621	9.0437	0.9737	2.8939
LOS(2nd Order)	3.1040	6.3737	0.9557	12.4707
Aguirre L.[13]	27.2242	48.8617	0.4955	86.0420
Bansal J *et al.*[14]	3.1301	9.8472	0.9543	18.6056

Table IV Comparison Of Reduced Models

Method	Reduced Models	ISE
Differentiation Method	$\frac{15.8672\, s + 11.4793}{16.042\, s^2 + 32.172\, s + 12}$	0.0661
Aguirre L. [13]	$\frac{0.2211\, s + 0.0702}{s^2 + 1.9531\, s + 0.1415}$	6.0352
Bansal J. *et al.* [14]	$\frac{1.869\, s + 0.5585}{s^2 + 2.663\, s + 0.5838}$	0.0494

V. CONCLUSIONS

The present work deals with the reduced order modelling of large scale SISO systems. The HOS have been reduced by differentiation technique and the performance is compared with other order reduction methods already available in the literature. The transient response parameters such as; rise time, settling time, peak and peak time of both HOS and LOS have been compared. It has been observed that reduced order system obtained using differentiation is more appreciable as compared to the ROMs already available in the literature for the same HOS. The step and frequency responses of HOS and LOS have also been compared and it is found that response of LOS obtained by differentiation is effectively closer to the HOS.

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