A SYNTHETIC STUDY TO VALIDATE THE ALGORITHM 3DINV DEVELOPED FOR INTERPRETATION OF THREE DIMENSIONAL MAGNETOTELLURIC DATA

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ABSTRACT

The magnetotelluric (MT) method is a very useful technique for studying the geoelectrical structure of earth. For resolving the subsurface structure this method requires an efficient modeling algorithm. Here we present an algorithm for forward and inverse modeling of three dimensional (3-D) MT data. Comparison of forward modeling is done using published results. Synthetic model for inversion has been purposely chosen to highlight the application of MT methods in oil exploration.

Keywords*: *Forward modeling, Inversion, Magnetotelluric

I. INTRODUCTION

All electromagnetic methods are used to map the subsurface resistivity of earth from the surface measurements. A variety of these methods are used for mineral, hydrocarbon, geothermal explorations and estimation of basement for foundation work. Magnetotelluric is also an electromagnetic method which in contrast to other EM methods makes use of the naturally existing electromagnetic fields as source for probing the earth. In this technique the electromagnetic (EM) waves generated from the ionosphere are used to study the earth properties, viz. electrical resistivity or conductivity. In magnetotelluric method earth is considered as a plane at air-earth interface whereas EM waves considered as plane wave due to the fact that the distance of ionosphere (> 75 km) from the earth is very large.

In MT the depth of investigation is up to hundreds of kilometer as it uses low frequency natural EM fields (0.0001 Hz to several kHz) whereas in controlled source electromagnetic method (CSEM) the depth of investigation is up to few tens of kilometers. The use of natural EM fields as source and greater depth of investigation makes the MT technique cost effective in comparison to other EM methods. MT survey also has advantage over seismic reflection methods for hydrocarbon exploration in volcanic regions where the later method fails. Recently, MT surveys are being done in marine environment to compliment the other expensive EM survey techniques (Key et al., 2004, Zhdanov et al., 2004).

The data collected at surface is inverted to estimate the subsurface resistivity/conductivity structure. As earth is three-dimensional, a two-dimensional (2D) earth model is not the true representation of earth. This is a simple reason why we need 3-D inversion. Moreover various studies had been done to show that if data contains 3-D structures, 2-D inversion may lead to wrong interpretation (Siripunvaraporn et al., 2005b, Ledo 2006).

In this paper we present results ofthe developed algorithm**3DINV** for efficient3-D inversion for MT data implemented in parallel (OpenMP) using F90 programing language.

II. 3-D MT FORWARD MODELING

2.1 Governing Equation

The propagation and attenuation of EM fields is governed by Maxwell's equations. At low frequency range used in MT, the displacement currents are negligible in comparison to conduction currents. We have assumed time dependence as$e^{i\omega t}$. Using this the Maxwell's equations in frequency domain are:

$$\nabla \times \boldsymbol{E} = -i\omega\mu_0 \boldsymbol{H} \quad (1)$$

$$\nabla \times \boldsymbol{H} = \sigma_T \boldsymbol{E} \quad (2)$$

Using (1) and (2), a vector Helmholtz equation for ***E*** can written as:

$$\nabla \times \nabla \times \boldsymbol{E} + i\omega\mu_0\sigma_T \boldsymbol{E} = 0 \quad (3)$$

After computing electric fields magnetic fields are computed using (1).

2.2 Responses

In MT survey we measure horizontal electric fields (E_x , E_y) and both horizontal and vertical magnetic fields (H_x , H_y , H_z). The MT impedance tensor describes the linear relation between electric and magnetic field components. Using all horizontal components of both electric and magnetic field, the impedance tensor (***Z***) can be written as:

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{Bmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{Bmatrix} \begin{pmatrix} H_x \\ H_y \end{pmatrix} \quad (4)$$

As there are two orthogonal polarizations of sources, (4) can be written as:

$$\begin{pmatrix} E_{x1} & E_{x2} \\ E_{y1} & E_{y2} \end{pmatrix} = \begin{Bmatrix} Z_{xy} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{Bmatrix} \begin{pmatrix} H_{x1} & H_{x2} \\ H_{y1} & H_{y2} \end{pmatrix} \quad (5)$$

where subscripts 1 and 2 indicates two polarizations. Further the complex impedance tensor (***Z***) in (5) can be converted into apparent resistivity and phase as:

$$\rho_{ij} = \frac{1}{\omega\mu_0} \left| Z_{ij} \right|^2 \quad (6)$$

$$\emptyset_{ij} = tan^{-1}\{Imag(Z_{ij})/Real(Z_{ij})\} \quad (7)$$

2.3 Forward Solution

To solve (3) we used the finite difference method (FDM) after discretizing the model domain. We have used the staggered grid (Yee 1966) for this purpose. In staggered ***E*** is defined along cell edges and ***H*** is defined on the face centers (Fig. 1).

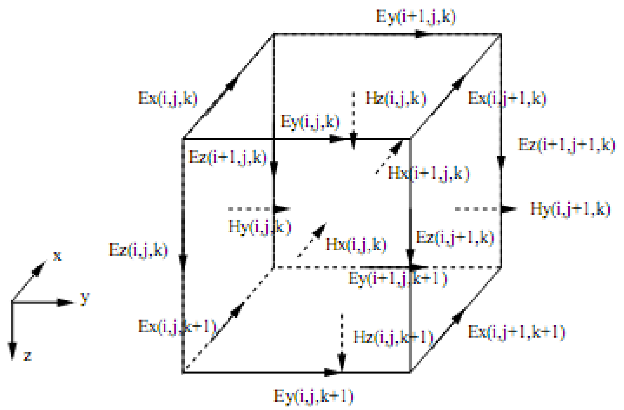


Figure1:staggered grid cell used for discretizing.

After discretizing the MT forward problem is transformed to a system of linear equations:

$$\mathbf{Ae = b} \tag{8}$$

where**A** is a highly sparse symmetric matrix (Fig. 2), **e** is a vector containing unknown internal electric fields, and **b** is obtained from boundary conditions. Now (8) is solved using iterative method bi-conjugate gradient stabilized (BI-CGSTAB). To improve the convergence rate of iterative solver DILU preconditioner is also used. Moreover to improve convergence at low frequencies static divergence correction (Smith 1996a, 1996b) is also applied.

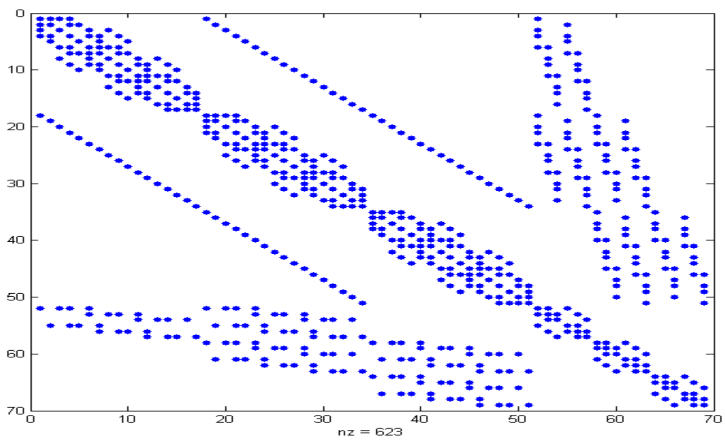


Figure 2: structure of matrix A associated with grid (4*3*4).

III. 3-D MT INVERSION

In inversion the objective is to find the 'smoothest' model subjected to a reasonable fit to the data (Constable el al.,1987). This objective is achieved by minimizing the following functional $U(\mathbf{m}, \lambda)$:

$$U(m,\lambda) = (m - m_0)^T C_m^{-1}(m - m_0) + \lambda^{-1}\{(d - F[m])^T C_d^{-1}(d - F[m])\} \quad (9)$$

where m is the resistivity model, m_0 is the initial guess model, C_m is the model covariance matrix, d is the observed data, $F[m]$ is the model response, C_d is the data covariance matrix and λ^{-1} is the Lagrange multiplier. This optimization problem is nonlinear because model response is a nonlinear function of model parameters. I have solved this optimization problem using an iterative approach, which is based on quasi linearizing the problem as:

$$F[m_{k+1}] = F[m_k + \Delta m] = F[m_k] + J_k(m_{k+1} - m_k) \quad (10)$$

where the subscript k in the equation is the iteration number, J_k is the jacobian matrix. By using (9) and (10), we get new model parameter in (k+1)th iteration as:

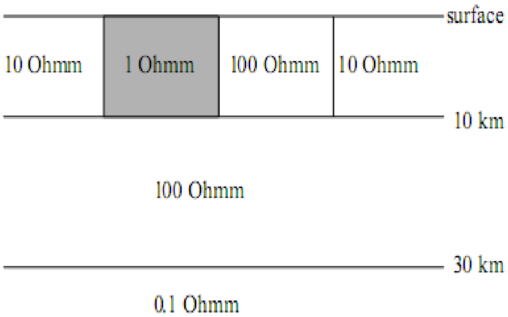
$$m_{k+1}(\lambda) = [J_k^T C_d^{-1} J_k + \lambda C_m^{-1}]^{-1} J_k^T C_d^{-1} \hat{R}_k + m_0 \quad (11)$$

where $\hat{R}_k = (d - F[m_k]) + J_k(m_k - m_0)$ and finally (11) is solved using conjugate gradient method.

IV. RESULTS

4.1 Validation of Forward Algorithm

To validate the developed forward modeling algorithm, response for a standard synthetic model is calculated. This model is used by various researchers (Siripunvaraporn et al., 2002, Mackie et al., 1993) to validate their algorithm. It consists of two rectangular blocks, of which one is conducting and other is resistive in the top layer of a three layer model (Fig. 3).



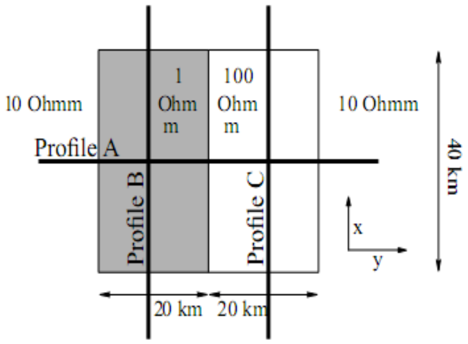


Figure 3: cross-section (top) view of the synthetic model and profiles where data is recorded (bottom).

The response is computed along three profiles A, B and C (Fig. 3).The comparison of results (response) shown in Fig. 4a, 4b and 4c confirms the accuracy of the developed forward modeling algorithm.

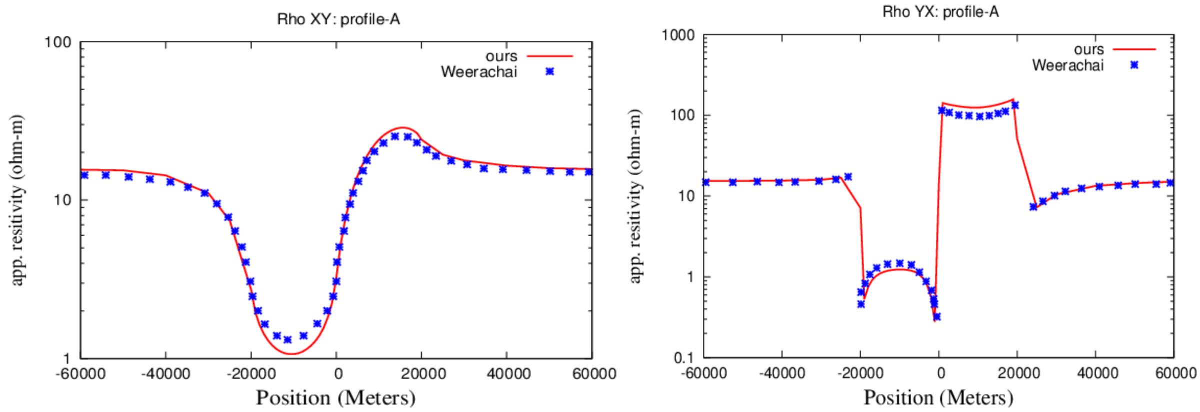


Figure 4a: comparison of response along profile A

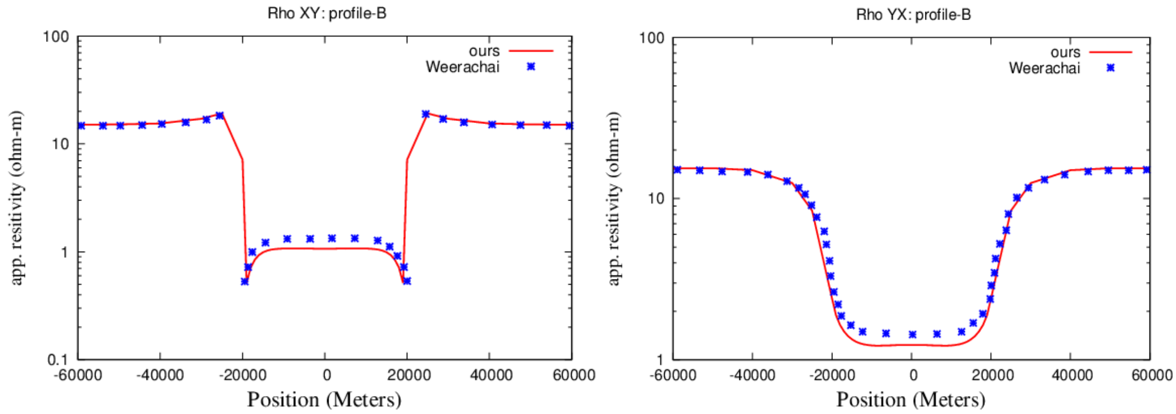


Figure 4b: comparison of response along profile B

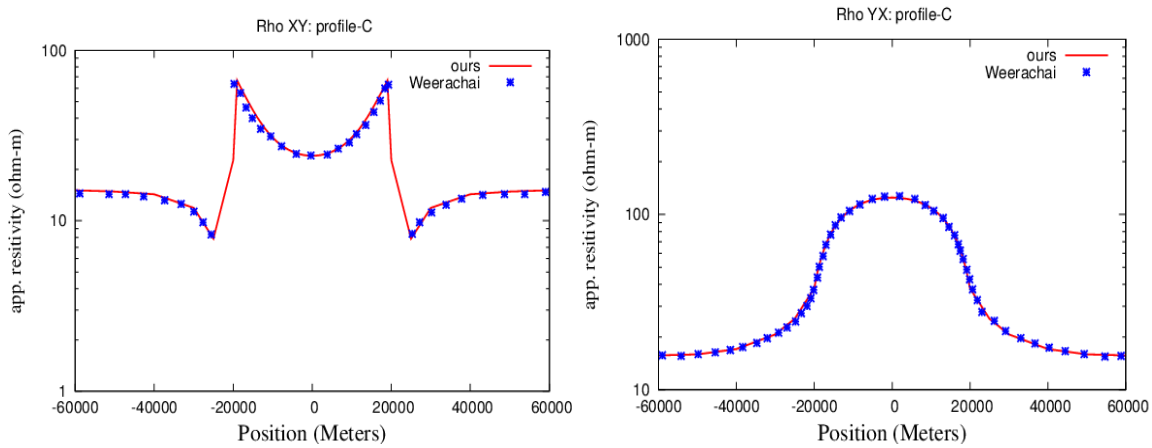


Figure 4c: comparison of response along profile C.

4.2 Inversion

To illustrate the developed MT inversion algorithm**3DINV**a synthetic model (Fig. 5) is used which is loosely inspired by Kelbert et al., (2014). Model consists of a folded structure of resistivity 1 ohm-m is buried inside a half space of resistivity 100 ohm-m. Data is recorded at 182 sites (Fig. 5) for 5 frequencies between 0.1Hz to 1000 Hz.

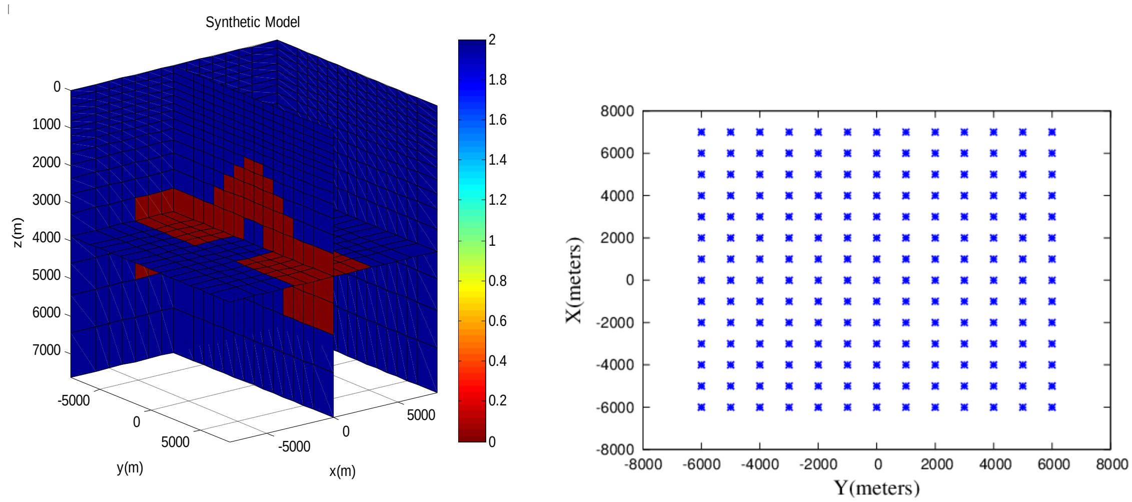


Figure 5: left: synthetic model ($log\rho$ is plotted) and right: layout of station's locations.

For inversion 2% Gaussian noise is added in the data and a homogeneous model of 100 Ωm resistivity is used as initial guess, this model is also used as prior model (m_0). Model produced after inversion is shown in Fig. 7.

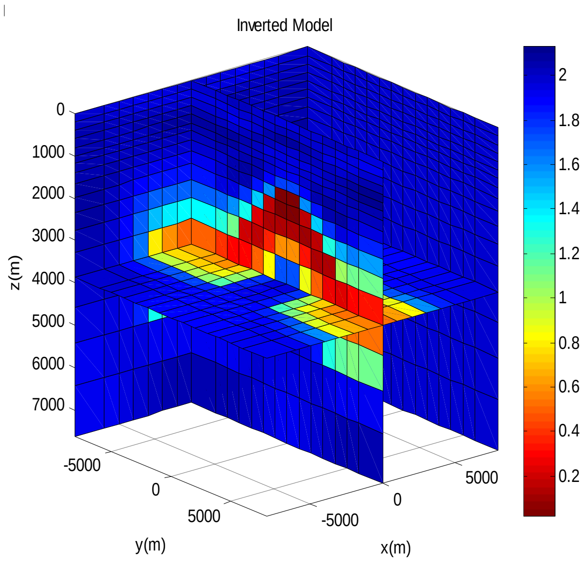


Figure 7: inverted model ($log\rho$ is plotted).

V. CONCLUSION

In the present paper we have presented an algorithm**3DINV** for forward and inverse modeling of 3-D magnetotelluric data. The benchmarking of forward modeling is done using published results. To illustrate the capability of MT inverse modeling we have considered a synthetic case of a fold model. Such structures are common in oil exploration where MT methods are used in conjecture with other exploration methods. The result shows that algorithm **3DINV** is able of resolving such structure and is capable of inverting real data. .

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